Fairness, Efficiency and Flexibility in Organ Allocation for Kidney Transplantation

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Abstract

We propose a scalable, data-driven method for designing national policies for the allocation of deceased donor kidneys to patients on a waiting list, in a fair and efficient way. We focus on policies that have the same form as the one currently used in the U.S. In particular, we consider policies that are based on a point system, which ranks patients according to some priority criteria, *e.g.*, waiting time, medical urgency, etc., or a combination thereof. Rather than making specific assumptions about fairness principles or priority criteria, our method offers the designer the flexibility to select his desired criteria and fairness constraints from a broad class of allowable constraints. The method then designs a point system that is based on the selected priority criteria, and approximately maximizes medical efficiency, *i.e.*, life year gains from transplant, while simultaneously enforcing selected fairness constraints.

Using our method, we design a point system that has the same form, uses the same criteria and satisfies the same fairness constraints as the point system that was recently proposed by U.S. policymakers. In addition, the point system we design delivers an 8% increase in extra life year gains. We evaluate the performance of all policies under consideration using the same statistical and simulation tools and data as the U.S. policymakers use. We perform a sensitivity analysis which demonstrates that the increase in extra life year gains by relaxing certain fairness constraints can be as high as 30%.

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1. Introduction

Renal or kidney transplantation and maintenance dialysis are the only treatments for *end-stage renal disease* (ESRD), a terminal disease affecting over $500,000$ people currently in the United States, see USRDS (2009). Despite being a major surgical procedure, transplantation is the treatment of choice for ESRD patients, as a successful transplantation improves their quality of life. In particular, dialysis treatment requires that the patient visits a dialysis center for at least 12 hours each week, whereas transplantation typically allows the patient to resume regular life activities. Furthermore, a multitude of research and clinical studies have statistically demonstrated that transplantation also reduces the mortality risk for patients, see Suthanthiran and Strom (1994), Schnuelle et al. (1998), Port et al. (1993), Ojo et al. (1994). Thus, a kidney transplant is considered by many as a potentially life-saving gift.

The two sources of kidneys for transplantation are living donors (*e.g.*, family members or friends of the patient) and deceased or cadaveric donors. The majority of patients are unsuccessful in finding living donors, and thus join a pool of patients waiting for a deceased donor organ. Of course, while in the living donor case the donation is typically made to a specific patient, in the deceased donor case an important allocation problem arises. In particular, once an organ is procured from a deceased donor, there can be thousands of medically compatible and available recipients the organ can be allocated to. The problem becomes even more significant, if one accounts for the organ shortage and the size of the pool of waiting patients in the United States: On October 20th 2010, $86,391$ patients were waiting for a kidney transplant. In 2009, there were $33,671$ new additions, but only $16,829$ transplantations were performed, from which $10,442$ transplants were from deceased donors. For more information and statistical details we refer the reader to UNOS (2010).

In recognition of the aforementioned allocation problem and the growing difficulty of matching supply and demand, the U.S. Congress passed the *National Organ Transplant Act* (NOTA) in 1984. According to this legislation, deceased donor organs are viewed as national resources in the U.S., and as such, their allocation has to be based on fair and equitable policies. Moreover, the sale of organs as well as money transfers of any nature in the acquisition of organs are strictly prohibited. Instead, the policy for allocating the organs should utilize waiting lists and have the form of a *priority method*. That means that patients in need of a transplant register on waiting lists. Then, once an organ is procured, all medically compatible patients are ranked according to some priority rules and the organ is successively offered to them according to their ranking, until it is accepted by a patient. Subsequent to the NOTA, the U.S. Congress established in 1984 the *Organ Procurement and Transplantation Network* (OPTN) in order for it to maintain a national registry for organ matching and develop allocation policies.

Naturally, the aforementioned allocation policies are of central importance and have to accomplish major objectives in alleviating human suffering, prolonging life and providing nondiscriminatory, fair and equal access to organs for all patients, independent of their race, age, blood group or other peculiar physiological characteristics. Some of the main challenges in designing a kidney allocation policy are the following:

- *Fairness constraints*: What does fair and equal access to organs mean? Due to the subjective nature of fairness, there is no single fairness criterion that is universally accepted by policymakers and academics alike. As such, a great challenge lies in identifying the appropriate fairness constraints that the allocation outcomes of a policy should ideally satisfy. An example of such a constraint could be a lower bound on the percentage of organs allocated to a particular group of patients – say, requiring that at least 47% of all transplants are received by recipients of blood type O. In the absence of such a constraint these groups would otherwise be handicapped and not have access to organs because of their physiological characteristics. A number of such criteria have been studied by OPTN policymakers (see OPTNKTC (2008), RFI (2008)).

- *Efficiency*: As a successful transplantation typically prolongs the life of a patient, while also improving his quality of life, the policy needs to ensure that the number of quality adjusted life year gains garnered by transplantation activities is as high as possible. This is also in line with the view of organs as national resources. Again, this objective is of paramount importance to the current policy design OPTNKTC (2008).

- *Prioritization criteria*: The policy needs to be based on medically justified criteria and physiological characteristics of patients and organs in order to rank patients. However, ethical rules disallow the use of criteria that can be deemed as discriminatory (*e.g.*, race, gender, etc.).

- *Simplicity*: Patients need to make important decisions about their treatment options, together with their physicians. To this end, they need to be able to estimate the probability of receiving an organ, or at least understand the allocation mechanism. For that reason, the priority method that is used needs to be simple and easy to communicate.

- *Implementation*: Suppose that one has selected his desired fairness constraints, prioritization criteria and a simple priority method. How does he then balance the emphasis put on the different prioritization criteria, so as to design a policy whose allocation outcomes would maximize efficiency, while satisfying the fairness constraints?

All the above challenges were faced by the OPTN policymakers in 2004, when they initiated the development of a new national allocation policy that will eventually replace the current one. In 2008, the OPTN released a concrete proposal in a *Request for Information* publication (RFI (2008)) that is currently under consideration by the U.S. Department of Health and Human Services.

In this work, we deal with the implementation challenge in designing a national allocation policy, while accounting for all the other challenges above. In particular, we focus on perhaps the simplest, most common and currently in use priority method, namely a point system. We make the following contributions:

1. We present a novel method for designing allocation policies based on point systems in a systematic, data-driven way. Our method offers the flexibility to the policymaker to select the fairness constraints he desires, as well as the prioritization criteria on which the point system will be based on. The method then outputs a conforming point system policy that approximately maximizes medical efficiency, while satisfying the fairness constraints.

2. We use our method to design a policy that (a) matches the fairness constraints of the recently proposed policy by U.S. policymakers, and (b) is based on the same criteria and simple scoring rule format. Critically though, it achieves an 8% increase in anticipated extra life year gains, as demonstrated by our numerical simulations, which are based on the statistical and simulation tools currently in use by U.S. policymakers (see below).

3. We use our method to perform a sensitivity analysis that explores the consequences from relaxing or introducing fairness constraints – for instance, what is the impact of reducing the

percentage of transplants to patients on dialysis for greater than 15 years by 1%? In the case of some constraints, relaxations of fairness constraints can result in life year gains on the order of 30%. As such, we believe this is a tool of great value in the policy design process.

Performance in all our numerical studies is evaluated using the same statistical and simulation tools, as well as data, as the U.S. policymakers use. Those tools and datasets were obtained directly from their developers, namely the *United Network for Organ Sharing* (UNOS), which is the non-profit organization that operates the OPTN, and the *Scientific Registry of Transplant Recipients* (SRTR).

The structure of this paper is as follows. In the next subsection, we review relevant work in the literature. Section 2 provides background information on the distribution of organs, the current allocation policy, as well as updates on the recent development of a new proposed policy. In Section 3, we discuss our method for designing allocation policies in detail. Section 4 includes numerical evidence of the usefulness of our work through the design of a new policy, the evaluation of its performance via simulation and a sensitivity analysis. We conclude in Section 5. A list of acronyms used appears at the end of this paper.

1.1. Literature Review

The model-based analysis of the organ allocation process has attracted significant interest in the academic literature. One of the first papers in this vain is by Ruth et al. (1985), in which the authors develop a simulation model to study the problem. Righter (1989) and David and Yechiali (1995) formulate the problem as a stochastic assignment problem and analyze stylized models that fit into that framework. Zenios et al. (2000) introduce a fluid model approximation of the organ allocation process that allows them to explicitly account for fairness and medical efficiency in the allocation.

Another stream of research focuses on the decision-making behavior of patients, by dealing with organ acceptance policies. David and Yechiali (1985) model the candidate's problem as an optimal stopping problem. Similar acceptance policies are developed by Ahn and Hornberger (1996) and Howard (2002). The present paper will test policies on a simulator developed by SRTR for OPTN; this simulator assumes a specific, exogenous acceptance model for patients built from historical data. While the acceptance model ignores endogeneity it allows us to simulate outcomes in precisely the manner policy makers currently do.

Recent work by Su and Zenios (2004) and Su and Zenios (2005) attempts to combine the above streams of research by explicitly accounting for the acceptance behavior of patients in the development of an allocation policy. In a similar vein, Su and Zenios (2006) propose an allocation mechanism that elicits the utilities of the patients. For more details, we refer the reader to the thorough review by Zenios (2005).

In all the above referenced work dealing with organ allocation policies, the authors design general near optimal dynamic policies. These papers take the important perspective of designing a *fundamentally new* allocation system from the ground up. In our work, we restrict our attention to policies that comply with the precise constraints imposed by current practice. That is, we focus our attention on policies based on simple point systems of the precise format as the ones currently in use and proposed by U.S. policymakers. Moreover, instead of designing a particular policy, we develop a framework that admits various fairness constraints and prioritization criteria. In other words, we design a mechanism that can fit directly in the *current* decision-making process of the U.S. policymakers.

2. Distribution and Allocation Policies

In this section, we briefly review the distribution process and the operation of the UNOS/OPTN as coordinators and developers of national policies for the allocation of deceased donor kidneys to patients. We then discuss the requirements such policies need to meet, and focus on policies that are based on point systems or scoring rules. Finally, we review the current policy in use in the U.S. (which itself is based on a scoring rule), as well as updates on the development of a new scoring rule based national policy.

In the U.S., the non-profit *Organ Procurement Organizations* (OPOs) are directly responsible for evaluating, procuring and allocating donated organs within their respective designated service area. Once consent is obtained and an organ is procured by an OPO, the OPTN computerized national registry automatically generates a list of patients who are medically compatible with the procured organ. Medical compatibility of patients is determined by the physiological characteristics they are listed with and those of the procured organ (*e.g.*, accounting for ABO incompatibility[1],

[1] ABO incompatibility is a reaction of the immune system that occurs if two different and not compatible blood types are mixed together, see `http://www.nlm.nih.gov/medlineplus/ency/article/001306.htm`.

weight and size, unacceptable antigens, etc.). Subsequently, the priority method used by the OPO determines the order in which the organ will be offered to patients. Once a kidney is procured, it can be typically preserved for up to 36-48 hours, after which the organ can no longer be used for transplantation. For that reason, priority is given to local patients, although there are rules that determine when priority should be given to non local patients. After an offer is being made to a patient, he has to decide with his surgeon whether to accept it or not within a limited amount of time. In case of rejection, the organ is offered to the next patient according to the specified order and so on. In case no patient accepts the organ within 36-48 hours, the organ is discarded.

In addition to using the OPTN national registry, the activities of the OPOs, and their allocation policies in particular, are coordinated and regulated by the OPTN. That is, the OPTN provides general guidelines and lays out a national allocation policy that is suggested to all OPOs. The allocation policy of every OPO then needs to be consistent with the national policy, although minor alterations are possible subject to approval by the OPTN.

2.1. National Allocation Policies

National policies for the allocation of the deceased donor kidneys are developed by the OPTN *Kidney Transplantation Committee* (KTC), and are approved by the U.S. Department of Health & Human Services. Policies need to account for numerous legal, economic, institutional, ethical, and other societal factors; the requirements for an allocation policy are included in the OPTN Final Rule (DHHS (2000)). Below we summarize the most important guidelines that policies have to conform to as per the OPTN Final Rule. In particular, the allocation

1. Shall seek to achieve the best use of donated organs, and avoid organ wastage;
2. Shall set priority rankings based on sound medical judgment;
3. Shall balance medical efficiency (extra life years) and equity (waiting time), without discriminating patients based on their race, age and blood type;
4. Shall be reviewed periodically and revised as appropriate.

Additionally, the priority method in place needs to be simple and easy to communicate, as discussed in the Introduction. As such, the ranking of patients is typically achieved by means of a *point system* or *scoring rule*: all national allocation policies that have been used in practice have been based on scoring rules. We formally define next the notion of a scoring rule based policy and then discuss the current national policy and suggested revisions.

Point system or Scoring rule based policies. Under a policy based on a scoring rule, patients are ranked according to a calculated score, commonly referred to in this context as the *Kidney Allocation Score* (KAS). Specifically, a scoring rule consists of *score components* and scalar constant *score weights*. A score component can be any function of the characteristics of a patient and/or an organ. Then, once an organ is procured and needs to be allocated, one calculates the individual score components for each patient and the particular procured organ. The KAS for each patient is evaluated as the weighted sum of his score components (using the score weights). To introduce some notation, given a patient p and an organ o, we denote the jth score component with $f_{j,(p,o)}$, and the jth score weight with w_j. The KAS of patient p for receiving organ o, $\text{KAS}(p, o)$, is then calculated as

$$\text{KAS}(p, o) = \sum_j w_j f_{j,(p,o)}.$$

For instance, examples of score components can be the number of years the patient has been registered on the waiting list for, the life expectancy of the patient in case he remained on dialysis, or the life expectancy in case he received the procured organ, etc.

One can think of a scoring rule based policy as a priority method that awards points to patients based on different criteria (the score components); patients are also potentially awarded different amounts of points per criterion, based on the score weights. The ranking is then achieved based on the number of points collected by each patient. The current policy in use and the one recently proposed by U.S. policymakers are both examples of scoring rule based policies and are discussed next.

Current allocation policy. The current policy has been in existence for more than 20 years. It is based on a scoring rule that utilizes waiting time, a measure of the patient's sensitization[2] and tissue matching[3] of the organ and the patient as score components. The rationale behind

[2]Potential recipients are "sensitized" if their immune system makes antibodies against potential donors. Sensitization usually occurs as a consequence of pregnancy, blood transfusions, or previous transplantation. Highly sensitized patients are more likely to reject an organ transplant than are unsensitized patients. For more information, see `http://www.ustransplant.org/`

[3]When two people share the same human leukocyte antigens (abbreviated as HLA), they are said to be a "match", that is, their tissues are immunologically compatible with each other. HLA are proteins that are located on the surface of the white blood cells and other tissues in the body. For more information, see

this rule is as follows. Points are given for waiting time and sensitization in order to serve the fairness objective of the allocation and to provide equal access to organs to all patients (note that highly sensitized patients have reduced medical compatibility with donors). On the other hand, since tissue matching is an indication for a successful transplantation, the points given to matched patients serve the medical efficiency objective of the allocation. For more details we refer the reader to ODADK (2010).

Recent advances in medicine and changes in patients' needs have rendered the current policy inappropriate. More specifically, these changes have rendered the current policy inconsistent with the OPTN Final Rule, see Norman (2009) and RFI (2008). For instance, the long waiting times experienced by the patients, coupled with advances in medicine that have prolonged the survivability of patients on dialysis, have resulted in the accumulation of points for waiting time by the patients. This accumulation of points has then created an imbalance between the efficiency and fairness objectives of the allocation, see OPTNKTC (2007). In response to that, and in line with the requirement of the OPTN Final Rule for periodic review of the policy, the KTC has been reviewing the policy for the past few years and is currently in the process of developing a new policy, see OPTNKTC (2007).

Development of a new policy. The OPTN has set the primary objective of the new policy to be the design of a scoring rule that strikes the right balance between fairness and medical efficiency. As mentioned above, the design of a scoring rule involves the identification of the appropriate score components, and the corresponding score weights that form the Kidney Allocation Score. The KTC has selected to base the score components on the following criteria. For a patient p and an organ o, the criteria are

1. *Life years from transplant* $\mathrm{LYFT}(p,o)$, which is equal to the expected incremental quality-adjusted life years gain of patient p from receiving organ o, compared to remaining on dialysis (for a precise definition, we refer the reader to Wolfe et al. (2008));

2. *Dialysis time* $\mathrm{DT}(p)$, which is equal to the years the patient has already spent on dialysis;

3. *Donor profile index* $\mathrm{DPI}(o)$, which is a number between 0 and 1, indicating the quality of the donated organ (0 corresponds to an organ of highest quality);

http://www.stanford.edu/dept/HPS/transplant/html/hla.html

4. *Calculated panel reactive antibody* $\mathrm{CPRA}(p)$, which is a number between 0 and 100, measuring the sensitization of the patient (0 corresponds to the lowest sensitization level).

This selection is currently being reviewed by the Office for Civil Rights of the U.S. Department of Health & Human Services for approval, see UNOS (2010).

Meanwhile, the KTC considered more than 28 different scoring rules based on the above criteria, and utilized simulation to evaluate their performance and identify the appropriate weights (see OPTNKTC (2008)). The dominant proposal up to this point, published in 2008 in a Request For Information document (RFI (2008)), entails the following formula for the Kidney Allocation Score:

$$\begin{aligned}\mathrm{KAS}(p,o) = &\, 0.8\,\mathrm{LYFT}(p,o) \times (1-\mathrm{DPI}(o)) \\ &+ 0.8\,\mathrm{DT}(p) \times \mathrm{DPI}(o) \\ &+ 0.2\,\mathrm{DT}(p) \\ &+ 0.04\,\mathrm{CPRA}(p).\end{aligned}$$

The rule is comprised of four components. The first two components are the life years from transplant and dialysis time, scaled by the donor profile index. The scaling ensures that in case of a high quality organ (DPI close to 0), emphasis is given on life years from transplant, whereas in case of a low quality organ (DPI close to 1), emphasis is given on dialysis time. The last two components are the dialysis time and calculated panel reactive antibody score of the patient. More information and motivating aspects can be found within the Request For Information document (RFI (2008)). As an example, consider an organ o of medium quality, with $\mathrm{DPI}(o) = 0.55$. Then, patients are awarded $0.8 \times (1 - 0.55) = 0.36$ points for every quality adjusted incremental life year they would gain in expectation, $0.8 \times 0.55 + 0.2 = 0.64$ points for every year they have spent on dialysis, and 0.04 points for every point of their CPRA score.

While medical expertise and the OPTN Final Rule can guide the identification of the appropriate score components, the task of finding the right weights is more involved, as the experimentation of the OPTN KTC with more than 28 different rules suggests. A natural question in response to the proposed scoring rule is whether this is the best we can do. In particular, does there exist another scoring rule of the same simple format that dominates the proposed one, *i.e.*, is equally or more fair and more efficient? In all fairness, this is an involved question to answer; to illustrate that, consider

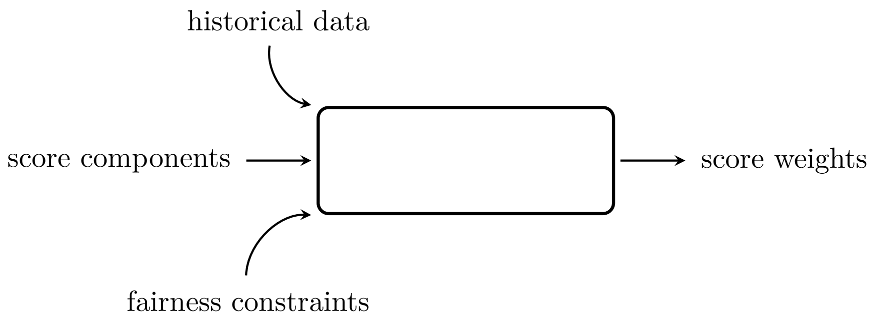

Figure 1: An illustration of the functionality of the proposed method.

only changing the weights in the proposed scoring rule above. The outcomes by such a change can perhaps be evaluated only via simulation; simulating a single specific scoring rule takes hours. Our proposed methodology makes a step towards answering those questions and is discussed next.

3. Designing Allocation Policies

We propose a method for designing scoring rule based policies for the allocation of deceased donor kidneys to patients. Specifically, we propose a data-driven method that computes in a systematic way score weights associated to pre-specified score components, so that the resulting policy achieves a near-optimal medical utility (measured by life years from transplant gains). In other words, after one has decided upon the components he wishes to include in a scoring rule, our method utilizes historical data to efficiently compute associated weights, so as to maximize the efficiency of the policy. In addition, our method can also take as input fairness constraints on the allocation outcomes; while we defer the precise definition of the class of admissible constraints for Section 3.1, we point here that our method captures a multitude of important and commonly studied constraints of interest to policymakers. Then, the method computes the score weights, so that the resulting policy is as efficient as possible, and the fairness constraints are approximately satisfied.

Figure 1 illustrates the functionality of the proposed method. Typically, policymakers select their desired score components that would feature in the scoring rule and constraints that the allocation outcomes need to satisfy. Our method provides an efficient, scalable and systematic way of striking the right balance between the selected score components by designing a policy that approximately maximizes medical efficiency, subject to the selected constraints.

As an application of our method, we use historical data from 2008, to construct a scoring rule based policy that utilizes the same criteria for components as the current proposal by the OPTN Kidney Transplantation Committee. We also ensure that the resulting policy has similar fairness characteristics with the KTC proposal. Numerical studies then suggest that the policy constructed by our method achieves an 8% improvement in life years from transplant, using the same statistical and simulation tools and data as U.S. policymakers use. Furthermore, we perform a trade-off analysis by considering deviations from the fairness constraints of the proposed policy. In particular, we study the effect in life year gains of the policy, in case of emphasizing or deemphasizing the priority given to patients who have been waiting for a long time or are sensitized. Our method efficiently redesigns the policy accordingly. The results indicate that the performance gain in life years from transplant can be as high as 30% in that case. Details on the application of our method and simulation studies are included in Section 4.

We next present our proposal in full detail.

3.1. Methodology

Given a list of n score components, related historical data of patients' and donated organs' characteristics, and constraints on the allocation outcomes (precisely defined below), we calculate score weights $w_1, \ldots, w_n$, such that the resulting scoring rule policy satisfies the constraints approximately, while maximizing life years from transplant.

Consider a fixed time period over which we have complete (ex facto) information about all patients registered in the waitlist (pre-existing and arriving) in that time period. In particular, we know their physiological characteristics, the time of their initial registration, as well as the evolution of their medical status and availability for a transplant during that time period. Suppose we also have complete information about the organs that are procured during the period, that is the time at which they are procured and their physiological characteristics. We index the patients by $p = 1, \ldots, P$ and the organs by $o = 1, \ldots, O$. We say that patient p is *eligible* to receive organ o, or equivalently that the patient-organ pair (p, o) is eligible for transplantation, if at the time of the organ procurement all conditions below are met:

1. The patient is registered at the waitlist for a transplant;
2. The patient is actively waiting for a transplant and his medical status is appropriate for transplantation;

3. The patient is medically compatible with the organ.

Let $\mathcal{C}$ be the set of patient-organ pairs eligible for transplantation, *i.e.*,

$$\mathcal{C} = \{(p,o) : \text{ patient } p \text{ is eligible to receive } o\}.$$

Note that one can construct $\mathcal{C}$ simply by using the arrival information and characteristics of the organs and the patients, and the evolution of the availability and medical status of the patients.

Additionally, one can also compute the score components for each eligible patient-organ pair, as well as the life years from transplant. Let $f_{j,(p,o)}$ be the value of the jth component score, $j = 1, \ldots, n$, and LYFT(p,o) the life years from transplant for pair $(p,o) \in \mathcal{C}$.

We now define the class of admissible constraints on the allocation outcomes, alluded to thus far. First, let $x_{(p,o)}$ be defined for every eligible patient-organ pair (p,o) as

$$x_{(p,o)} = \begin{cases} 1, & \text{if organ } o \text{ is assigned to patient } p, \\ 0, & \text{otherwise.} \end{cases}$$

A constraint is admissible for our method if it is *linear*, that is if it can be modeled as a linear constraint with respect to variable x. The class of constraints that can be modeled in this way is very broad, and captures the majority of constraints a social planner might wish to incorporate; for instance, one can impose lower bounds for a specific group of patients on

- the probability of receiving a transplant,
- the average life years from transplant gained among the actual transplant recipients,
- the average time spent on dialysis among the actual transplant recipients.

As an example, a lower bound L on the number of organs allocated to a specific group of patients $\mathcal{G} \subset \{1, \ldots, P\}$, can be expressed as

$$\sum_{p \in \mathcal{G}} \sum_{o:(p,o) \in \mathcal{C}} x_{(p,o)} \geq L;$$

for instance setting $\mathcal{G}$ to be the set of all patients of blood type O could enforce a lower bound on transplants for patients of this blood type.

We denote the input fairness constraints with $Ax \leq b$, for some matrix A and vector b.

We now present our method. Consider a social planner with foresight who has knowledge of the set of all eligible pairs $\mathcal{C}$ and the life years from transplant score for every pair in the set. Suppose also that patients accept all organs offered to them. In this setup, the problem of allocating organs to patients so as to maximize medical efficiency, *i.e.*, life years from transplant, subject to fairness constraints $Ax \leq b$, can be formulated as a linear optimization problem:

$$
\begin{aligned}
\text{maximize} \quad & \sum_{(p,o)\in\mathcal{C}} \mathrm{LYFT}(p,o)x_{(p,o)} \\
\text{subject to} \quad & \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} \leq 1, \quad \forall p \\
& \sum_{p:(p,o)\in\mathcal{C}} x_{(p,o)} \leq 1, \quad \forall o \\
& Ax \leq b \\
& x \geq 0.
\end{aligned} \tag{1}
$$

Note that a fractional value for $x_{(p,o)}$ can be interpreted as the probability of assigning organ o to patient p in a randomized policy.

By linear optimization duality, if y is the vector of optimal dual multipliers associated with the constraints $Ax \leq b$ for problem (1), then problem (1) is equivalent with the one below:

$$
\begin{aligned}
\text{maximize} \quad & \sum_{(p,o)\in\mathcal{C}} \mathrm{LYFT}(p,o)x_{(p,o)} - y^T Ax + y^T b \\
\text{subject to} \quad & \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} \leq 1, \quad \forall p \\
& \sum_{p:(p,o)\in\mathcal{C}} x_{(p,o)} \leq 1, \quad \forall o \\
& x \geq 0.
\end{aligned} \tag{2}
$$

Note that problem (2) is a matching problem. We equivalently rewrite the objective of (2) as $c^T x + y^T b$, utilizing the cost vector c defined as

$$
c_{(p,o)} = \mathrm{LYFT}(p,o) - \left(y^T A\right)_{(p,o)}, \quad \forall (p,o) \in \mathcal{C}.
$$

Note that our goal is to design a policy that approximately solves the above matching problem online, *i.e.*, a policy that sequentially matches organs at their time of procurement to available patients without utilizing any future information. One possible way of achieving that is by greedily matching procured organs to patients based on the coefficients c. However, those coefficients are calculated above utilizing all information available. Moreover, our goal is to rank patients not by any artificial score coefficients, but rather based on the selected score components. To this end, one can calculate the appropriate score weights, such that the linear combination of the score components based on them is as close as possible to the coefficients c. Specifically, the score weights $w_1, \ldots, w_n$ are found by running a standard linear regression, with the values of the score components for each eligible patient-organ pair as independent variables, and the coefficients of c as dependent variables. That is, we compute the weights such that for every eligible patient-organ pair,

$$c_{(p,o)} \approx w_0 + w_1 f_{1,(p,o)} + \ldots + w_n f_{n,(p,o)}.$$

The method is summarized as Procedure 1.

Procedure 1 Computation of score weights

Input: list of n score components, data for linear constraints (A, b), historical data: set of eligible patient-organ pairs $\mathcal{C}$, life years for transplant $\text{LYFT}(p,o)$ and values of score components, $f_{j,(p,o)}$, $j = 1, \ldots, n$, for every eligible pair (p,o).

Output: weights for scoring rule, $w_1, \ldots, w_n$.

1: solve problem (1)
2: $y \leftarrow$ vector of optimal dual multipliers associated with constraints $Ax \leq b$
3: $c_{(p,o)} \leftarrow c_{(p,o)} = \text{LYFT}(p,o) - \left(y^T A\right)_{(p,o)}, \quad \forall (p,o) \in \mathcal{C}$
4: use linear regression to find $w_0, w_1, \ldots, w_n$, such that for all $(p,o) \in \mathcal{C}$

$$c_{(p,o)} \approx w_0 + w_1 f_{1,(p,o)} + \ldots + w_n f_{n,(p,o)}.$$

3.2. Discussion

In this section, we discuss (a) why one should expect the proposed method to perform well in practice, and (b) the relative merits of our contribution.

Consider the airline network revenue management setting analyzed in Talluri and van Ryzin (1998). In that setting, an airline is operating flights and is selling different itinerary tickets to incoming customers, so as to maximize net expected profits from sales subject to capacity constraints (which correspond to the numbers of seats on the different aircrafts operating the flights). The authors analyze a simple control policy that decides whether to sell an itinerary ticket to a passenger or not, and demonstrate that the policy is asymptotically optimal under some conditions. For the organ allocation problem, a simplified version of the policy that we described in the previous section can be cast in the same framework as in Talluri and van Ryzin (1998); one can then derive a similar result of asymptotic optimality, following the same procedure. In particular, in Talluri and van Ryzin (1998), the authors analyze the performance of the following simple bid-price control policy: one first solves a capacity allocation problem assuming that demand is deterministic and equal to the mean demand. Based on the optimal dual multipliers associated with the resource capacity constraints in that problem, one then calculates a "bid price" for every unit of a particular resource. An itinerary ticket is then sold to a customer if the money offered by the customer exceed the sum of prices of the resources he would consume. In our procedure, if we ignore the regression step, we also assume deterministic demand and solve a similar allocation problem[4]. We then calculate "bid prices" y associated to the fairness constrains and assign the organ to the patient who achieves the highest profit (LYFT), adjusted for the "bid prices". For more details, we refer the reader to the paper by Talluri and van Ryzin (1998).

Apart from the above discussion regarding the performance of our method in practice, we next provide numerical evidence. Before that, we summarize the relative merits of our contribution.

1. The proposed method uses detailed historical medical data to extract near optimal score weights in an efficient manner. In particular, the method is highly scalable and can learn the parameters from potentially highly detailed and complicated historical datasets, with no need for simplifications, clustering or grouping of patients' and/or organs' characteristics.

2. The method offers the flexibility and allows policymakers to focus only on identifying score components and desired fairness properties of allocation outcomes in the design of a new policy. The method undertakes the more involved part of finding the appropriate weights and balancing the score components. Although medical intuition can help in making educated guesses for the weights, there is little guarantee that a policy designed in such way would yield the desired results. Furthermore, even if a set of weights yields a policy with

[4]Specifically, consider the deterministic linear optimization model analyzed in Talluri and van Ryzin (1998), where the different customer classes correspond to patient classes, the profits correspond to life years from transplant and the network capacity constraints correspond to the fairness constraints. If we instead use historical samples rather than averages, we recover formulation (1).

the desired outcomes, there can be another policy delivering a superior performance. Due to the computational intensity of simulations, one simply cannot explore all possible combinations of weights. Our contribution is towards this direction, by using mathematical tools to automatically extract near-optimal weights from historical data.

3. In this work, we develop our method in the context of kidney allocation. However, the same procedure can be generalized in a straightforward manner for other organs as well. Thus, our methodology is particularly useful for any organ allocation policy one wishes to design based on scoring rules.

4. The failure of the current kidney allocation policy in place to keep up with advances in medicine and the changes in patients' needs throughout the years, has demonstrated that in such a dynamic and complex environment, revisions to policies are likely to be required in the future as well, a fact that is also recognized by the OPTN Final Rule. Furthermore, even in the current process of developing a new policy, there is no guarantee that the Office of Civil Rights will approve the criteria of life years from transplant, dialysis time, etc., suggested by the OPTN policymakers. In both cases, our method will expedite the development of a new policy, as it would require only an updated list of score components and fairness properties to be specified.

5. Our method allows for sensitivity analysis; specifically, one can efficiently evaluate the outcomes of relaxing some or introducing new fairness constraints. In the next section, we provide such an analysis that reveals the dependence of medical efficiency on fairness concepts, and illustrate how it can be used in practice by policymakers. In particular, note that one of the main goals that the OPTN policymakers have set for a new national policy is to deemphasize the role of waiting time and increase medical efficiency (see Section 2.1). Our analysis provides a characterization of the trade-offs involved.

In the next section, we provide numerical evidence of the usefulness of the described method. In particular, we use historical data to create a new scoring policy that performs better than the one proposed by the Kidney Transplantation Committee and also explore other options by means of a sensitivity analysis.

4. Numerical Evidence and the Design of a New Allocation Policy

We utilize the method described in the previous section to design a new scoring-rule based allocation policy. We set as benchmark the dominant proposal of the OPTN Kidney Transplantation Committee (referred to also as the KTC policy in this section), presented in the Request for Information document in 2008 (see Section 2). We design a policy that uses the same criteria as score components and achieves an 8% increase in life years from transplant, while exhibiting similar fairness properties. Finally, we perform a trade-off analysis by considering deviations from the fairness properties of the proposed policy. To ensure a fair comparison, we evaluate the performance of the policies by using the same statistical models and tools, as well as datasets with the OPTN KTC policymakers. We first provide details about the data and models, and then present our methodology and results.

4.1. Data, Statistical Models and Tools

This work uses highly detailed historical data from the Scientific Registry of Transplant Recipients (SRTR). The SRTR data system includes data on all donor, wait-listed candidates, and transplant recipients in the U.S., submitted by the members of the Organ Procurement and Transplantation Network (OPTN). The datasets include all the various physiological and demographic characteristics of wait-listed patients and donors that are needed for our study, as well as the evolution of the medical status of the patients, and the arrival process of the donated organs.

In addition, the SRTR has developed sophisticated survivability models for ESRD patients using historical survival rates. The models provide an estimate for the anticipated lifespan of a patient in case he remained on dialysis, or in case he received a particular kidney, based on a plethora of physiological attributes (*e.g.*, the patient's age, body mass index, diagnosis, as well as tissue matching, the donor's age, cause of death, etc.). For more information and a detailed study of the statistical performance of the models, we refer the reader to Wolfe et al. (2008) and Wolfe et al. (2009). The SRTR has also developed an acceptance model that predicts the probability of a particular patient accepting a particular organ offered to him, based on the physiological characteristics of the patient and the donor, the distance, etc.

The above datasets and statistical models have also been utilized by the SRTR in the development of the *Kidney-Pancreas Simulated Allocation Model* (KPSAM). The KPSAM is an event-driven simulator that simulates the entire allocation process using historical data, for different allocation policies. It was developed in order to support studies of alternative policies. The KP-SAM is the platform that the OPTN KTC is utilizing to evaluate the performance of their proposed policies, see OPTNKTC (2007). For more details on the data and the simulator, we refer the reader to Waisanen et al. (2004) and KPSAM (2008).

For the purposes of this study, we obtained the KPSAM and utilized its simulation engine in order to obtain realistic allocation outcomes of the policies we consider. The life years from transplant gains are estimated using the aforementioned survivability models, embedded in the KPSAM.

4.2. Methodology

Using the KPSAM we simulate the KTC policy for the 2008 dataset. We record the number of transplantations occurring and the net life years from transplant. To explore the fairness properties of the policy, we record the percentage distribution of transplant recipients across different races, age groups, blood types, sensitization groups, as well as diagnosis types, years spent on dialysis and geographical regions. Note that this practice is in line with the comparison criteria studied by the OPTN policymakers (see OPTNKTC (2008), RFI (2008)).

To design a new policy based on our method described in Section 3, we use the following as input:

- Historical data: We use the first 6 months of data of the 2008 dataset as input to our method (training data). The data pertaining to the remaining 6 months is used to evaluate performance.

- Score components: We use the life years from transplant (LYFT), dialysis time (DT) [5] and calculated panel reactive antibody (CPRA) as the score components. Note that the components are based on exactly the same criteria as in the KTC policy [6]. In summary, the scoring rule rewards the patient with w_1 points per life year from transplant gained, w_2 per year on dialysis for the first 5 years, w_3 per year on dialysis for years 5-10 and w_4 per year beyond

[5] Our scoring rule is piece-wise linear in this component.

[6] The Donor Profile Index (DPI) component proves superfluous.

the 10th, and w_5 points per percentage point of CPRA.

- Fairness constraints: To ensure that the fairness properties are similar to the KTC policy, we use the recorded percentage distributions (see above) for the KTC policy as input constraints. We use lower bound constraints on the percentage of organs allocated to the following groups: Caucasian, African-American, Hispanic or patients of another race; patients aged between 18-34, 34-49, 49-64 and above 64 years; patients who have spent less than 5, 5-10, 10-15 or more than 15 years on dialysis; blood type O, A, B, AB patients; patients diagnosed with nephritis, hypertension, polycystic kidney disease, diabetes or other disease; patients with a sensitization level (CPRA) of 0-10, 10-80 or 80-100; patients registered at each of the 11 distinct geographical regions[7] in the U.S. in which UNOS operates. For instance, consider the fairness constraints pertaining to dialysis time. The recorded percentage distribution of recipients for the KTC policy is as follows: 55.4% of the recipients have spent less than 5 years on dialysis, 28.8% between 5-10 years, 10.2% between 10-15 years and 5.6% more than 15 years. The constraints we add then as input to our method are:

$$\begin{aligned}
&\sum_{p:\, 0 \le \mathrm{DT}(p) \le 5} \ \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} \ge \frac{55.4}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)},\\
&\sum_{p:\, 5 \le \mathrm{DT}(p) \le 10} \ \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} \ge \frac{28.8}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)},\\
&\sum_{p:\, 10 \le \mathrm{DT}(p) \le 15} \ \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} \ge \frac{10.2}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)},\\
&\sum_{p:\, \mathrm{DT}(p) \ge 15} \ \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} \ge \frac{5.6}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)}.
\end{aligned} \tag{3}$$

To evaluate the performance of the method, we use the KPSAM to simulate the output policy for the 6 months of 2008 that were not used as input. We record the number of transplantations occurring and the net life years from transplant. To compare the fairness properties of the policy, we also record the same percentage distributions of transplant recipients as for the KTC policy (see above).

[7] For its own operational purposes, UNOS has divided the U.S. into 11 distinct geographical regions. Region 1 for instance includes all the states in New England. For more information, see `http://www.unos.org/docs/Article_IX.pdf`.

4.3. Results

The output of the method is the scoring rule assigning the Kidney Allocation Score to a patient-organ pair (p, o) of

$$\mathrm{KAS}(p,o) = \mathrm{LYFT}(p,o) + g\left(\mathrm{DT}(p)\right) + 0.12\,\mathrm{CPRA}(p),$$

where

$$g\left(\mathrm{DT}\right) = \begin{cases} 0.55\,\mathrm{DT}, & 0 \leq \mathrm{DT} \leq 5, \\ 2.75 + \mathrm{DT}, & 5 \leq \mathrm{DT} \leq 10, \\ 7.75 + 0.25\,\mathrm{DT}, & 10 \leq \mathrm{DT}. \end{cases}$$

According to the above scoring rule, patients are awarded 1 point for every life year from transplant gain, 0.12 points per point of their sensitization score and points based on their dialysis time as follows: 0.55 points for the first 5 years, 1 point for every additional year up to 10 years and 0.25 points for every additional year beyond that.

We use simulation to compare the performance of the above designed policy and the KTC proposed policy for 6 months in 2008 (see Methodology above). The simulation results are presented in Table 1.

Compared to the KTC proposed policy, the one designed by our method delivers an almost identical performance in terms of percentage distributions of transplant recipients, but results in an important 8.2% increase in life years from transplant. Both policies appear to have the same performance in number of transplantations. Figure 2 illustrates the distribution of recipients under the two policies across the 11 distinct geographical regions that UNOS has divided the U.S. into. Again, in comparison with the KTC policy, the designed policy has an almost identical performance in terms of percentage distributions of transplant recipients across all regions.

In comparing the policies further, consider the organ recipients who spent less than 5 years on dialysis prior to receiving their transplant and the organ recipients who spent more than 5 years. Through the scaling of the LYFT and DT components with the donor profile index (DPI), the KTC policy directs better quality organs to patients with a higher LYFT score, whereas organs of marginal quality are offered to patients who spent many years on dialysis, as discussed in Section 2.1; for more details see OPTNKTC (2008), RFI (2008). As a result, one might expect that under

	KTC policy	Designed policy
transplantations and efficiency		
number of transplantations	$5,746$	$5,796$
net life years from transplant	$34,290$	$37,092$
avg life years from transplant	5.95	6.4
racial distribution		
caucasian	41.8%	43.5%
african-american	35.7%	33.3%
hispanic	13.9%	14.5%
other	8.6%	8.7%
age distribution		
18-34 yrs	5%	4.2%
34-49 yrs	26.4%	24.8%
49-64 yrs	50.6%	52.8%
64+ yrs	18%	18.2%
dialysis time distribution		
0-5 yrs	55.4%	56%
5-10 yrs	28.8%	28%
10-15 yrs	10.2%	10.1%
15+ yrs	5.6%	5.9%
blood type distribution		
O	47.9%	47.7%
A	37.9%	37.6%
B	11.7%	12%
AB	2.5%	2.7%
diagnosis type distribution		
nephritis	19.5%	18.9%
hypertension	21.6%	19.2%
polycystic	9.9%	11.8%
other	23%	25%
diabetes	26%	25.1%
sensitization level distribution		
CPRA 0-10	55.4%	54.9%
CPRA 10-80	24.7%	24.9%
CPRA 80+	19.9%	20.2%

Table 1: Simulation results of the KTC policy and the policy designed in Section 4, for an out-of-sample period of 6 months in 2008.

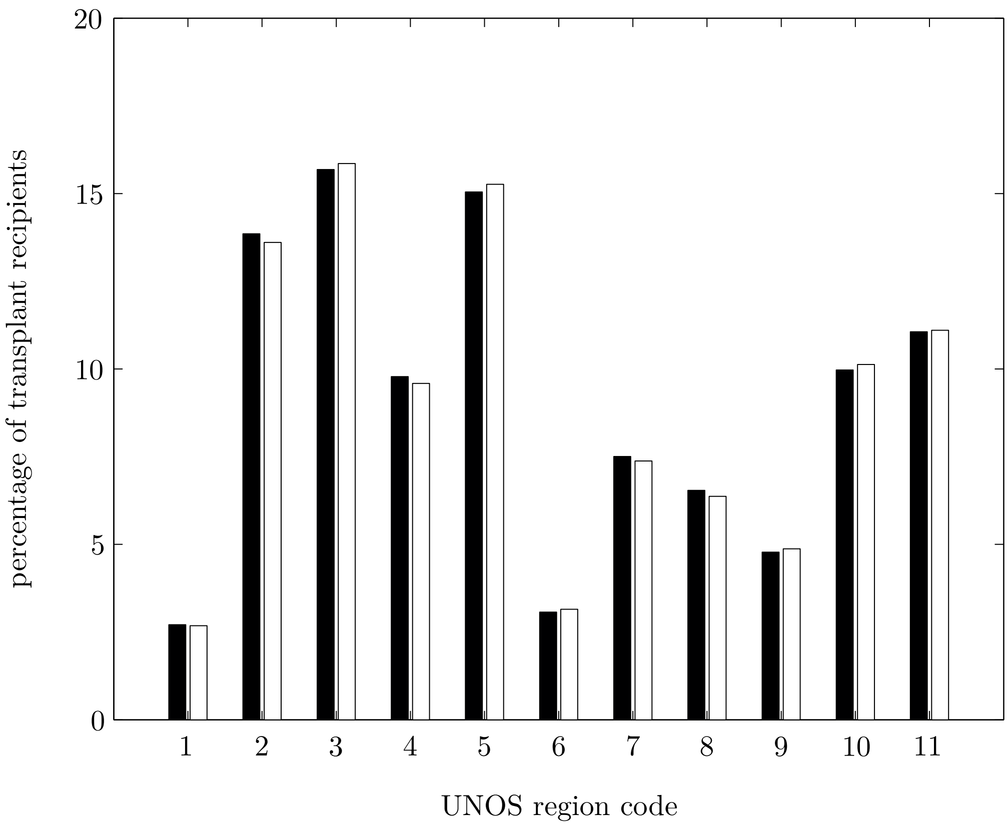

Figure 2: Simulated percentage distribution of transplant recipients across the 11 distinct geographical regions that UNOS has divided the U.S. into, under the KTC policy (white) and the policy designed in Section 4 (black), for an out-of-sample period of 6 months in 2008.

the KTC policy the organ recipients who spent less than 5 years on dialysis to be systematically allocated organs of better quality compared to those who spent more than 5 years. This is indeed reflected by our simulation results: the average life years from transplant gain of recipients who spent less than 5 years on dialysis is 14.2% higher than the average gain of all recipients; in contrast, the average life years from transplant gain of recipients who spent more than 5 years is 19.4% smaller than the average gain of all recipients. Under our policy however, the gain differences across those two groups are smaller: the differences are 9.7% higher than the average and 13.4% smaller than the average respectively for the two groups. That demonstrates that our policy provides a more equitable distribution of the organs, at least in that sense.

4.4. Sensitivity Analysis

We conclude our numerical experiments by demonstrating how our method can be used to perform a sensitivity analysis with respect to imposed fairness constraints. Similarly, one can perform an analysis with respect to changes in the score components used.

Specifically, we explore the dependence of life years from transplant gains on the priority given for dialysis time and sensitization. To this end, we redesign the policy by using the same procedure and input as above, but by considering slightly modified fairness constraints. In particular, we firstly use all the constraints used above, but relax the constraints pertaining to patient groups of different dialysis time, *i.e.*, constraints (3). The relaxation is performed by introducing a slack parameter s in the percentage requirements of recipients of different groups, that is, the relaxed constraints take the form

$$\begin{aligned}
\sum_{p:\, 0\leq \mathrm{DT}(p)\leq 5} \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} &\geq \frac{55.4-s}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)},\\
\sum_{p:\, 5\leq \mathrm{DT}(p)\leq 10} \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} &\geq \frac{28.8-s}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)},\\
\sum_{p:\, 10\leq \mathrm{DT}(p)\leq 15} \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} &\geq \frac{10.2-s}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)},\\
\sum_{p:\, \mathrm{DT}(p)\geq 15} \sum_{o:(p,o)\in\mathcal{C}} x_{(p,o)} &\geq \frac{5.6-s}{100} \sum_{(p,o)\in\mathcal{C}} x_{(p,o)}.
\end{aligned} \tag{4}$$

Clearly, for $s = 0$ one would recover the policy that was designed previously. For $s > 0$, the requirement on matching the percentage distribution (with regard to patient groups of different dialysis time) achieved by the KTC policy is relaxed. Thus one should expect that policies designed with such relaxed requirements would achieve higher life years from transplant gains. Using our method, we design policies for various values of the slack parameter s and quantify how the gains in medical efficiency depend on deviations from the selected fairness constraints. Secondly, we follow the same procedure to examine the dependence of medical efficiency on the priority given to sensitized patients. We again use all the constraints as in the previous subsection, but this time relax the constraints pertaining to patient groups of different sensitization levels. The relaxation is again performed using a slack parameter s. Note that one can potentially perform a sensitivity analysis though many other different ways of relaxing the constraints; for illustration purposes we focus here only on the described method of uniformly relaxing the constraints by a slack parameter.

The results we obtain in the aforementioned scenarios are depicted in Figures 3 and 4. Figure 3 shows the life years from transplant gains (for the 6 months period we consider) of policies designed with relaxed constraints on patient groups of different dialysis time, for various values of the slack parameter s. Similarly, Figure 4 shows the life years from transplant gains of policies designed with relaxed constraints on patient groups of different sensitization, for various values of the slack parameter s. The two figures also depict the operational points of the KTC proposed policy. Comparing the two, one can observe that the dependence of medical efficiency is stronger on dialysis time. Also, the life years from transplant gains can be as high as $44,300$ years, which are 30% larger than the gains of the KTC policy. Note that although such a policy might not be implementable, the analysis can provide insights to policymakers and facilitate their decision process.

5. Discussion and Future Directions

We dealt with the important problem of allocating deceased donor kidneys to waitlisted patients, in a fair and efficient way. We focused on national allocation policies in the United States and the recent effort to revise the current policy in place.

Particularly, we studied allocation policies that are based on point systems; under those policies patients are awarded points according to some priority criteria, and patients are then prioritized by the number of points awarded. We identified the challenges in designing a point system, specifically

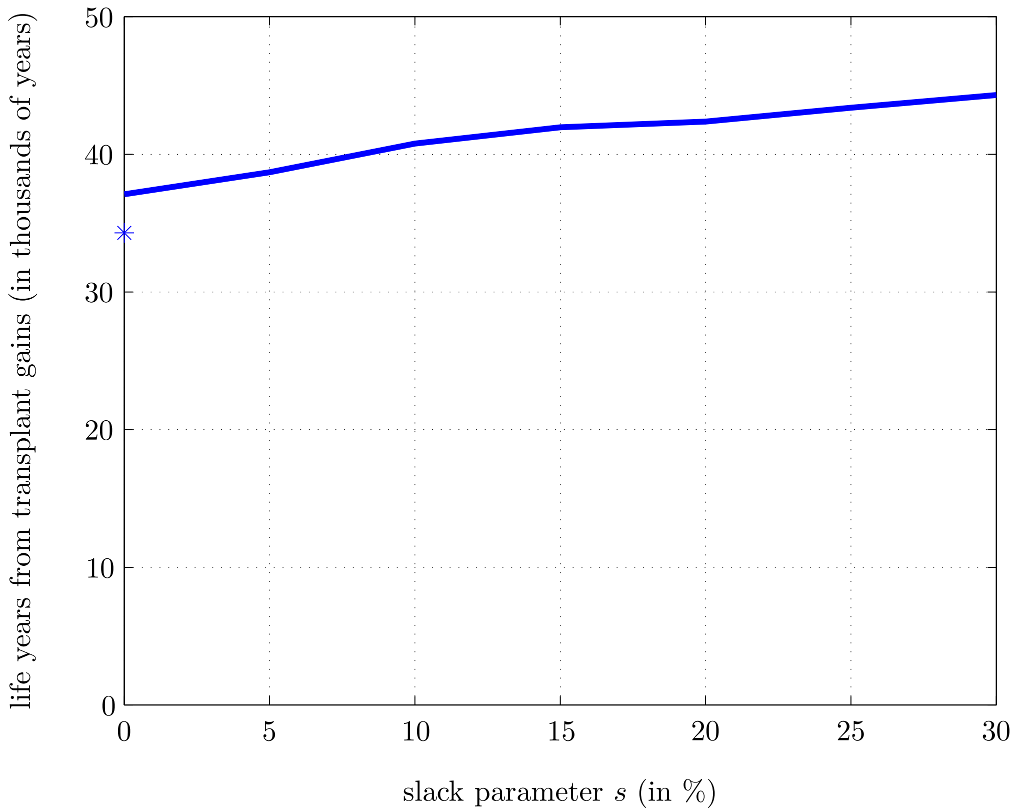

Figure 3: Simulated life years from transplant gains for policies (designed by our method) with relaxed constraints on all patient groups of different dialysis time, for various values of the slack parameter s; for more details see Section 4.4. The results are for an out-of-sample period of 6 months in 2008. The marker corresponds to the operational point of the policy proposed by the UNOS policymakers.

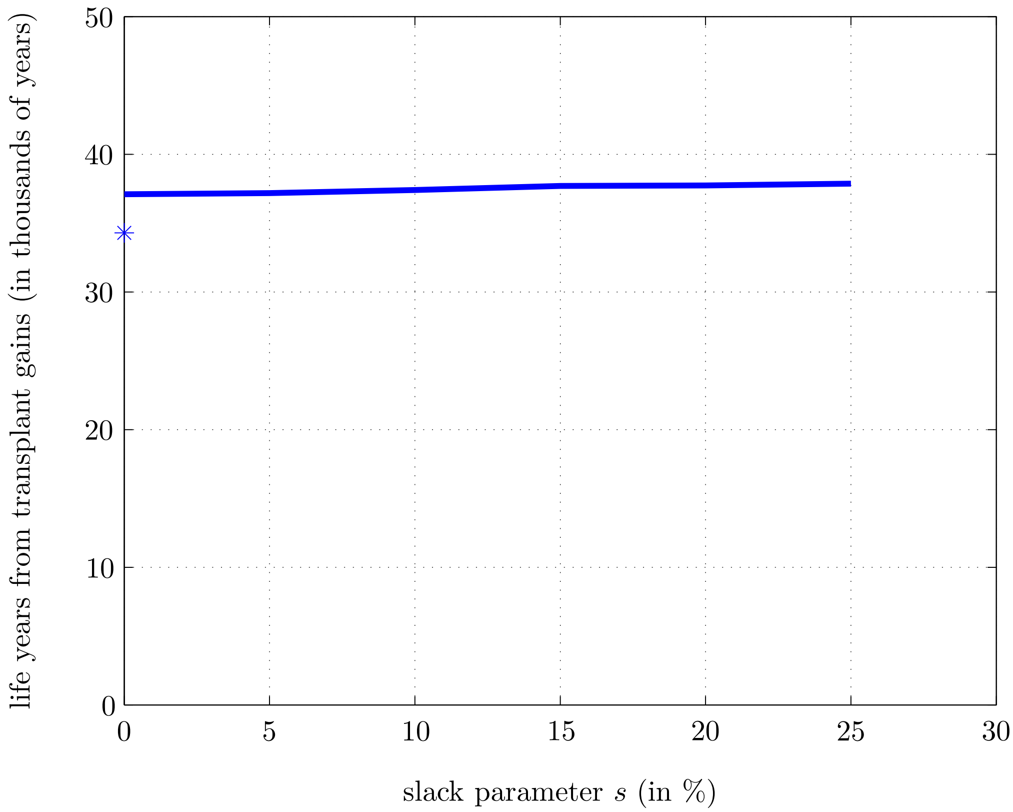

Figure 4: Simulated life years from transplant gains for policies (designed by our method) with relaxed constraints on all patient groups of different sensitization levels, for various values of the slack parameter s; for more details see Section 4.4. The results are for an out-of-sample period of 6 months in 2008. The marker corresponds to the operational point of the policy proposed by the UNOS policymakers.

the relative emphasis put on each criterion such that the resulting policy strikes the right balance between efficiency and fairness.

Our main contribution was a scalable, data-driven method of designing point system based allocation policies in an efficient and systematic way. The method does not presume any particular fairness scheme, or priority criterion. Instead, it offers the flexibility to the designer to select his desired fairness constraints and criteria under which patients are awarded points. Our method then balances the criteria and extracts a near-optimal point system policy, in the sense that the policy outcomes yield approximately the maximum number of life years gains (medical efficiency), while satisfying the fairness constraints.

Using our method, we designed a new policy that matches in fairness properties and priority criteria the policy that was recently proposed by the U.S. policymakers. Critically, our policy delivers an 8% relative increase in life years gains. The performance gain was established via simulation, utilizing the same statistical tools and data as the U.S. policymakers.

Finally, we presented a trade-off analysis that revealed the dependence of medical efficiency on the important fairness concepts of prioritizing patients who have either spent a lot of time waiting, or are medically incompatible with the majority of donors.

List of Acronyms

CPRA	Calculated Panel Reactive Antibody
DPI	Donor Profile Index
DT	Dialysis Time
ESRD	End-Stage Renal Disease
KAS	Kidney Allocation Score
KPSAM	Kidney-Pancreas Simulated Allocation Model
KTC	Kidney Transplantation Committee
LYFT	Life Years From Transplant
NOTA	National Organ Transplant Act
OPO	Organ Procurement Organization
OPTN	Organ Procurement and Transplantation Network
RFI	Request For Information
SRTR	Scientific Registry of Transplant Recipients
UNOS	United Network for Organ Sharing

Disclaimer

The data reported here have been supplied by the Arbor Research Collaborative for Health (Arbor Research) as the contractor for the Scientific Registry of Transplant Recipients (SRTR). The interpretation and reporting of these data are the responsibility of the authors and in no way should be seen as an official policy of or interpretation by the SRTR or the U.S. Government.

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